Solutions to short-answer questions

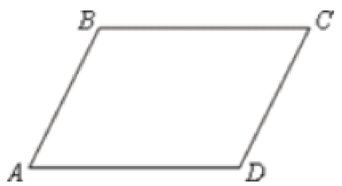
1 a $\boldsymbol{a}$ is parallel to $\boldsymbol{b}$ if $\boldsymbol{a} = k\boldsymbol{b}$, where k is a constant.

$$7\boldsymbol{i} + 6\boldsymbol{j} = k(2\boldsymbol{i} + x\boldsymbol{j})$$
$$2k = 7$$
$$k = \frac{7}{2}$$
$$kx = 6$$
$$\frac{7x}{2} = 6$$
$$x = \frac{12}{7}$$

b

$$|\boldsymbol{a}| = \sqrt{7^2 + 6^2}$$
$$= \sqrt{85}$$
$$|\boldsymbol{b}| = \sqrt{2^2 + x^2}$$
$$= |\boldsymbol{a}| = \sqrt{85}$$
$$\therefore\ x^2 + 4 = 85$$
$$x^2 = 81$$
$$x = \pm 9$$

2



$$A = (2, -1)$$
$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$
$$= 5\boldsymbol{i} + 3\boldsymbol{j}$$
$$B = (5, 3)$$
$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$
$$= \overrightarrow{AB} + \overrightarrow{AD}$$
$$= \boldsymbol{i} + 9\boldsymbol{j}$$
$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$$
$$= 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{i} + 9\boldsymbol{j}$$
$$= 3\boldsymbol{i} + 8\boldsymbol{j}$$
$$C = (3, 8)$$
$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$
$$= 4\boldsymbol{j}$$
$$D = (0, 4)$$

3 $\boldsymbol{a} + p\boldsymbol{b} + q\boldsymbol{c} = (2 + 2p - q)\boldsymbol{i} + (-3 - 4p - 4q)\boldsymbol{j} + (1 + 5p + 2q)\boldsymbol{k}$

To be parallel to the x-axis,

$$\boldsymbol{a} + p\boldsymbol{b} + q\boldsymbol{c} = k\boldsymbol{i}$$
$$1 + 5p + 2q = 0$$
$$2 + 10p + 4q = 0 \quad (1)$$
$$-3 - 4p - 4q = 0 \quad (2)$$
$$(1) + (2):$$
$$-1 + 6p = 0$$

$$p=\frac{1}{6}$$

$$1+\frac{5}{6}+2q=0$$

$$2q=-\frac{11}{6}$$

$$q=-\frac{11}{12}$$

4 a $\overrightarrow{PQ}=(3\boldsymbol{i}-7\boldsymbol{j}+12\boldsymbol{k})-(2\boldsymbol{i}-2\boldsymbol{j}+4\boldsymbol{k})$

$=\boldsymbol{i}-5\boldsymbol{j}+8\boldsymbol{k}$

$|\overrightarrow{PQ}|=\sqrt{1^2+5^2+8^2}$

$=\sqrt{90}=3\sqrt{10}$

b $\frac{1}{3\sqrt{10}}(\boldsymbol{i}-5\boldsymbol{j}+8\boldsymbol{k})$

5 $\overrightarrow{AB}=4\boldsymbol{i}+8\boldsymbol{j}+16\boldsymbol{k}$

$\overrightarrow{AC}=x\boldsymbol{i}+12\boldsymbol{j}+24\boldsymbol{k}$

For A, B and C to be collinear, we need

$\overrightarrow{AC}=k\overrightarrow{AB}$.

$x\boldsymbol{i}+12\boldsymbol{j}+24\boldsymbol{k}=k(4\boldsymbol{i}+8\boldsymbol{j}+16\boldsymbol{k})$

$8k=12$

$k=1.5$

$x=4k$

$=6$

6 a $\overrightarrow{OA}=\sqrt{4^2+3^2}$

$=5$

$\text{Unit vector}=\frac{1}{5}(4\boldsymbol{i}+3\boldsymbol{j})$

b $\overrightarrow{OC}=\frac{16}{5}\overrightarrow{OA}$

$=\frac{16}{5}\times\frac{1}{5}(4\boldsymbol{i}+3\boldsymbol{j})$

$=\frac{16}{25}(4\boldsymbol{i}+3\boldsymbol{j})$

7 a i $\overrightarrow{SQ}=\boldsymbol{b}+\boldsymbol{a}=\boldsymbol{a}+\boldsymbol{b}$

ii $\overrightarrow{TQ}=\frac{1}{3}\overrightarrow{SQ}$

$=\frac{1}{3}(\boldsymbol{a}+\boldsymbol{b})$

iii $\overrightarrow{RQ}=-2\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{a}=\boldsymbol{b}-\boldsymbol{a}$

iv $\overrightarrow{PT}=\overrightarrow{PQ}+\overrightarrow{QT}$

$=\overrightarrow{PQ}-\overrightarrow{TQ}$

$=\boldsymbol{a}-\frac{1}{3}(\boldsymbol{a}+\boldsymbol{b})$

$=\frac{1}{3}(2\boldsymbol{a}-\boldsymbol{b})$

v $\overrightarrow{TR} = \overrightarrow{TQ} + \overrightarrow{QR}$

$$= \overrightarrow{TQ} - \overrightarrow{RQ}$$
$$= \frac{1}{3}(\boldsymbol{a}+\boldsymbol{b}) - (\boldsymbol{b}-\boldsymbol{a})$$
$$= \frac{1}{3}(4\boldsymbol{a}-2\boldsymbol{b})$$
$$= \frac{2}{3}(2\boldsymbol{a}-\boldsymbol{b})$$

b $2\overrightarrow{PT} = \overrightarrow{TR}$

P, T and R are collinear.

8 $\boldsymbol{a} = \boldsymbol{b}$

a **i** $-s\boldsymbol{j} = 2\boldsymbol{j}$

$$s = -2$$

ii $5\boldsymbol{i} = t\boldsymbol{i}$

$$t = 5$$

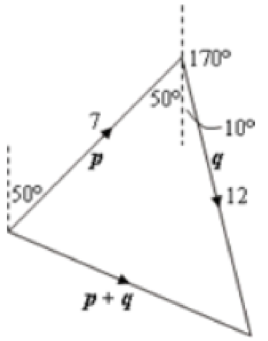
iii $2\boldsymbol{k} = u\boldsymbol{k}$

$$u = 2$$

b $\hat{\boldsymbol{a}} = \sqrt{5^2+2^2+2^2}$

$$= \sqrt{25+4+4}$$
$$= \sqrt{33}$$

9



Use the cosine rule

$$|\boldsymbol{p}+\boldsymbol{q}|^2 = 7^2 + 12^2 - 2\times 7\times 12\times \cos 60^\circ$$
$$= 109$$
$$|\boldsymbol{p}+\boldsymbol{q}| = \sqrt{109}$$

10a $\boldsymbol{a} + 2\boldsymbol{b} = (5\boldsymbol{i}+2\boldsymbol{j}+\boldsymbol{k}) + 2\times(3\boldsymbol{i}-2\boldsymbol{j}+\boldsymbol{k})$

$$= 11\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$$

b $|\boldsymbol{a}| = \sqrt{5^2+2^2+1^2}$

$$= \sqrt{30}$$

c $\hat{\boldsymbol{a}} = \frac{1}{\sqrt{30}}(5\boldsymbol{i}+2\boldsymbol{j}+\boldsymbol{k})$

d $\boldsymbol{a} - \boldsymbol{b} = (5\boldsymbol{i}+2\boldsymbol{j}+\boldsymbol{k}) - (3\boldsymbol{i}-2\boldsymbol{j}+\boldsymbol{k})$

$$= 2\boldsymbol{i} + 4\boldsymbol{j}$$

11a $\overrightarrow{OC} = \overrightarrow{OA} - \overrightarrow{OB}$

$$= (3\boldsymbol{i}+4\boldsymbol{j}) - (4\boldsymbol{i}-6\boldsymbol{j})$$

$$= -\boldsymbol{i} + 10\boldsymbol{j}$$
$$C = (-1, 10)$$

b $\boldsymbol{i} + 24\boldsymbol{j} = h(3\boldsymbol{i} + 4\boldsymbol{j}) + k(4\boldsymbol{i} - 6\boldsymbol{j})$

$3h + 4k = 1$

$4h - 6k = 24$

Multiply the first equation by 3 and the second equation by 2.

$9h + 12k = 3$ ①

$8h - 12k = 48$ ②

① + ②:

$$\begin{aligned} 17h &= 51 \\ h &= 3 \\ 9 + 4k &= 1 \\ k &= -2 \end{aligned}$$

12 $m\boldsymbol{p} + n\boldsymbol{q} = 3m\boldsymbol{i} + 7m\boldsymbol{j} + 2n\boldsymbol{i} - 5n\boldsymbol{j}$

$= 8\boldsymbol{i} + 9\boldsymbol{j}$

$3m + 2n = 8$

$7m - 5n = 9$

Multiply the first equation by 5 and the second equation by 2.

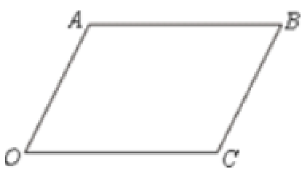
$15m + 10n = 40$ ①

$14m - 10n = 18$ ②

① + ②:

$$\begin{aligned} 29m &= 58 \\ m &= 2 \\ 6 + 2n &= 8 \\ n &= 1 \end{aligned}$$

13a



$$\begin{aligned} \boldsymbol{b} &= \overrightarrow{OB} \\ &= \overrightarrow{OA} + \overrightarrow{AB} \\ &= \overrightarrow{OA} + \overrightarrow{OC} \\ &= \boldsymbol{a} + \boldsymbol{c} \end{aligned}$$

b

$$\begin{aligned} \overrightarrow{AB} &= \boldsymbol{b} - \boldsymbol{a} \\ BC &= \boldsymbol{c} - \boldsymbol{b} \\ AB : BC &= 3 : 2 \\ \frac{AB}{BC} &= \frac{3}{2} \\ 2AB &= 3BC \\ 2(\boldsymbol{b} - \boldsymbol{a}) &= 3(\boldsymbol{c} - \boldsymbol{b}) \\ 2\boldsymbol{b} - 2\boldsymbol{a} &= 3\boldsymbol{c} - 3\boldsymbol{b} \\ 5\boldsymbol{b} &= 2\boldsymbol{a} + 3\boldsymbol{c} \\ \boldsymbol{b} &= \frac{2}{5}\boldsymbol{a} + \frac{3}{5}\boldsymbol{c} \end{aligned}$$

14 Let $\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j}$, $\boldsymbol{b} = -\boldsymbol{i} + 3\boldsymbol{j}$ and $\boldsymbol{c} = -2\boldsymbol{i} - 2\boldsymbol{j}$

a $\boldsymbol{a} \cdot \boldsymbol{a} = 13$

b $\boldsymbol{b} \cdot \boldsymbol{b} = 10$

c $\boldsymbol{c} \cdot \boldsymbol{c} = 8$

d $\boldsymbol{a} \cdot \boldsymbol{b} = -11$

e $\boldsymbol{a} \cdot (\boldsymbol{b} + \boldsymbol{c}) = (2\boldsymbol{i} - 3\boldsymbol{j}) \cdot (-3\boldsymbol{i} + \boldsymbol{j}) = -9$

f
$$\begin{aligned}(\boldsymbol{a} + \boldsymbol{b}) \cdot (\boldsymbol{a} + \boldsymbol{c}) &= \boldsymbol{a} \cdot \boldsymbol{a} + \boldsymbol{a} \cdot \boldsymbol{c} + \boldsymbol{b} \cdot \boldsymbol{a} + \boldsymbol{b} \cdot \boldsymbol{c} \\ &= 13 + 2 - 11 - 4 \\ &= 0\end{aligned}$$

g $\boldsymbol{a} + 2\boldsymbol{b} = 3\boldsymbol{j}$
$3\boldsymbol{c} - \boldsymbol{b} = -5\boldsymbol{i} - 9\boldsymbol{j}$
$\therefore (\boldsymbol{a} + 2\boldsymbol{b}) \cdot (3\boldsymbol{c} - \boldsymbol{b}) = -27$

15 $\overrightarrow{OA} = \boldsymbol{a} = 4\boldsymbol{i} + \boldsymbol{j}$

$\overrightarrow{OB} = \boldsymbol{b} = 3\boldsymbol{i} + 5\boldsymbol{j}$

$\overrightarrow{OC} = \boldsymbol{c} = -5\boldsymbol{i} + 3\boldsymbol{j}$

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= -4\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{i} + 5\boldsymbol{j} \\ &= -\boldsymbol{i} + 4\boldsymbol{j}\end{aligned}$$

$$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{BO} + \overrightarrow{OC} \\ &= -3\boldsymbol{i} - 5\boldsymbol{j} - 5\boldsymbol{i} + 3\boldsymbol{j} \\ &= -8\boldsymbol{i} - 2\boldsymbol{j}\end{aligned}$$

$\overrightarrow{AB} \cdot \overrightarrow{BC} = 8 - 8 = 0.$
Hence there is a right angle at B.

16 $\boldsymbol{p} = 5\boldsymbol{i} + 3\boldsymbol{j}$ and $\boldsymbol{q} = 2\boldsymbol{i} + t\boldsymbol{j}$

a If $\boldsymbol{p} + \boldsymbol{q}$ is parallel to $\boldsymbol{p} + \boldsymbol{q}$ there exists a non-zeo real number k such that.
$k(\boldsymbol{p} + \boldsymbol{q}) = \boldsymbol{p} - \boldsymbol{q}$.
That is,
$k(7\boldsymbol{i} + (3 + t)\boldsymbol{j} = 3\boldsymbol{i} + (3 - t)\boldsymbol{j}$.
Hence

$$\begin{aligned}7k &= 3 \\ k &= \frac{3}{7} \\ k(3 + t) &= (3 - t) \\ \therefore 3(3 + t) &= 7(3 - t) \\ \therefore 9 + 3t &= 21 - 7t \\ 10t &= 12 \\ t &= \frac{6}{5}\end{aligned}$$

b
$$\begin{aligned}\boldsymbol{p} - 2\boldsymbol{q} &= 5\boldsymbol{i} + 3\boldsymbol{j} - 2(2\boldsymbol{i} + t\boldsymbol{j}) \\ &= \boldsymbol{i} + (3 - 2t)\boldsymbol{j} \\ \boldsymbol{p} + 2\boldsymbol{q} &= 5\boldsymbol{i} + 3\boldsymbol{j} + 2(2\boldsymbol{i} + t\boldsymbol{j}) \\ &= 9\boldsymbol{i} + (3 + 2t)\boldsymbol{j}\end{aligned}$$

Since the vectors are perpendicular

$$\begin{aligned}(\boldsymbol{i} + (3 - 2t)\boldsymbol{j}) \cdot (9\boldsymbol{i} + (3 + 2t)\boldsymbol{j}) &= 0 \\ 9 + (3 - 2t)(3 + 2t) &= 0 \\ 9 + (9 - 4t^2) &= 0 \\ 4t^2 &= 18 \\ t^2 &= \frac{9}{2} \\ t &= \pm\frac{3}{\sqrt{2}}\end{aligned}$$

c $|\bmit{p} - \bmit{q}| = |3\bmit{i} + (3-t)\bmit{j}|$

$= \sqrt{9 + (3-t)^2}$

$|\bmit{q}| = |2\bmit{i} + t\bmit{j}|$

$= \sqrt{4 + t^2}$

If $|\bmit{p} - \bmit{q}| = |\bmit{q}|$

then $9 + (3-t)^2 = 4 + t^2$

$\therefore 9 + 9 - 6t + t^2 = 4 + t^2$

$14 - 6t = 0$

$t = \frac{7}{3}$

17 $\overrightarrow{OA} = \bmit{a} = 2\bmit{i} + 2\bmit{j}$

$\overrightarrow{OB} = \bmit{b} = \bmit{i} + 2\bmit{j}$

$\overrightarrow{OC} = \bmit{a} = 2\bmit{i} - 3\bmit{j}$

a i $\overrightarrow{AB} = -\bmit{a} + \bmit{b} = -\bmit{i}$

ii $\overrightarrow{AC} = -\bmit{a} + \bmit{c} = -5\bmit{j}$

b

$$\text{The vector resolute } = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\overrightarrow{AC} \cdot \overrightarrow{AC}} \overrightarrow{AC}$$

$$= 0$$

c 1

Solutions to multiple-choice questions

1 **C** $\boldsymbol{v} = \begin{bmatrix} 3-1 \\ 5-1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$a = 2, b = 4$

2 **C** $\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB}$

$= -\overrightarrow{AC} + \overrightarrow{AB}$

$= \boldsymbol{u} - \boldsymbol{v}$

3 **E** $\boldsymbol{a} + \boldsymbol{b} = \begin{bmatrix} 1+2 \\ -2+3 \end{bmatrix}$

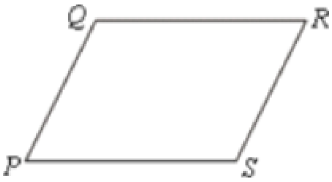
$= \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

4 **A** $2\boldsymbol{a} - 3\boldsymbol{b} = 2\begin{bmatrix} 3 \\ -2 \end{bmatrix} - 3\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 6 - -3 \\ -4 - 9 \end{bmatrix}$

$= \begin{bmatrix} 9 \\ -13 \end{bmatrix}$

5 **B**



$$\begin{aligned}\overrightarrow{SQ} &= \overrightarrow{SR} + \overrightarrow{RQ}\\ &= \overrightarrow{PQ} + -\overrightarrow{QR}\\ &= \boldsymbol{p} - \boldsymbol{q}\end{aligned}$$

6 **B** $$\begin{aligned}|3\boldsymbol{i} - 5\boldsymbol{j}| &= \sqrt{3^2 + (-5)^2}\\ &= \sqrt{9+25}\\ &= \sqrt{34}\end{aligned}$$

7 **A** $$\begin{aligned}\overrightarrow{AB} &= -\overrightarrow{OA} + \overrightarrow{OB}\\ &= (\boldsymbol{i} - 2\boldsymbol{j}) - (2\boldsymbol{i} + 3\boldsymbol{j})\\ &= -\boldsymbol{i} - 5\boldsymbol{j}\end{aligned}$$

8 **C** $$\begin{aligned}|\overrightarrow{AB}| &= |-\boldsymbol{i} - 5\boldsymbol{j}|\\ &= \sqrt{(-1)^2 + (-5)^2}\\ &= \sqrt{1+25}\\ &= \sqrt{26}\end{aligned}$$

9 **D** $$\begin{aligned}|\boldsymbol{a}| &= \sqrt{2^2+3^2}\\ &= \sqrt{13}\\ \hat{\boldsymbol{a}} &= \frac{1}{\sqrt{13}}(2\boldsymbol{i} + 3\boldsymbol{j})\end{aligned}$$

10 **C** $$\begin{aligned}|\boldsymbol{a}| &= \sqrt{3^2+1^2+3^2}\\ &= \sqrt{19}\\ \hat{\boldsymbol{a}} &= \frac{1}{\sqrt{19}}(-3\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k})\end{aligned}$$

Solutions to extended-response questions

1 $\begin{bmatrix}1\\0\end{bmatrix}$ is in the east direction and $\begin{bmatrix}0\\1\end{bmatrix}$ in the north direction.

a $$\begin{aligned}\overrightarrow{OP} &= -32\begin{bmatrix}0\\1\end{bmatrix} - 31\begin{bmatrix}1\\0\end{bmatrix}\\ &= \begin{bmatrix}-31\\-32\end{bmatrix}\end{aligned}$$

b The ship is travelling parallel to the vector $\boldsymbol{u} = \begin{bmatrix}4\\3\end{bmatrix}$ with speed 20 km/h.

The unit vector in the direction of $\boldsymbol{u}$ is $\frac{1}{5}\begin{bmatrix}4\\3\end{bmatrix}$.

$$\begin{aligned}\text{The vector } \overrightarrow{PR} &= \frac{20}{5}\begin{bmatrix}4\\3\end{bmatrix}\\ &= \begin{bmatrix}16\\12\end{bmatrix}\end{aligned}$$

The position vector ofthe ship is

$$\begin{aligned}\overrightarrow{OR} &= \overrightarrow{OP} + \overrightarrow{PR} \\ &= \begin{bmatrix} -31 \\ -32 \end{bmatrix} + \begin{bmatrix} 16 \\ 12 \end{bmatrix} \\ &= \begin{bmatrix} -15 \\ -20 \end{bmatrix} \\ &= -5\begin{bmatrix} 3 \\ 4 \end{bmatrix}\end{aligned}$$

c $$\begin{aligned}|\overrightarrow{OR}| &= 5\sqrt{3^2+4^2} \\ &= 25\end{aligned}$$

When the ship reaches R, it is $25\ \mathrm{km}$ from the lighthouse, and therefore the lighthouse is visible from the ship.

2 $\boldsymbol{p} = 3\boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{q} = -2\boldsymbol{i} + 4\boldsymbol{j}$

a $$\begin{aligned}\therefore |\boldsymbol{p} - \boldsymbol{q}| &= |3\boldsymbol{i} + \boldsymbol{j} - (-2\boldsymbol{i} + 4\boldsymbol{j})| \\ &= |5\boldsymbol{i} - 3\boldsymbol{j}| \\ &= \sqrt{25+9} \\ &= \sqrt{34}\end{aligned}$$

b $$\begin{aligned}|\boldsymbol{p}| &= \sqrt{9+1} \\ &= \sqrt{10} \\ \text{and } |\boldsymbol{q}| &= \sqrt{4+16} \\ &= 2\sqrt{5} \\ \therefore |\boldsymbol{p}| - |\boldsymbol{q}| &= \sqrt{10} - 2\sqrt{5}\end{aligned}$$

c $$\begin{aligned}3\boldsymbol{i} + \boldsymbol{j} + 2(-2\boldsymbol{i} + 4\boldsymbol{j}) + \boldsymbol{r} &= \boldsymbol{0} \\ 3\boldsymbol{i} + \boldsymbol{j} - 4\boldsymbol{i} + 8\boldsymbol{j} + \boldsymbol{r} &= \boldsymbol{0} \\ -\boldsymbol{i} + 9\boldsymbol{j} + \boldsymbol{r} &= \boldsymbol{0} \\ \text{Hence } \boldsymbol{r} &= \boldsymbol{i} - 9\boldsymbol{j}\end{aligned}$$

3 $\boldsymbol{a} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 11 \\ 7 \\ 3 \end{bmatrix}, \boldsymbol{c} = \begin{bmatrix} 7 \\ 9 \\ 7 \end{bmatrix}$ and $\boldsymbol{d} = \begin{bmatrix} 26 \\ 12 \\ 2 \end{bmatrix}$

a $$\begin{aligned}\boldsymbol{a} + 2\boldsymbol{b} - \boldsymbol{c} &= k\boldsymbol{d} \\ \therefore \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 11 \\ 7 \\ 3 \end{bmatrix} - \begin{bmatrix} 7 \\ 9 \\ 7 \end{bmatrix} &= k\begin{bmatrix} 26 \\ 12 \\ 2 \end{bmatrix} \\ \therefore \begin{bmatrix} 13 \\ 6 \\ 1 \end{bmatrix} &= k\begin{bmatrix} 26 \\ 12 \\ 2 \end{bmatrix}\end{aligned}$$

Therefore $k = \frac{1}{2}$ and $\boldsymbol{a} + 2\boldsymbol{b} - \boldsymbol{c} = \frac{1}{2}\boldsymbol{d}$

b $$\begin{aligned}x\boldsymbol{a} + y\boldsymbol{b} &= \boldsymbol{d} \\ \therefore x\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} + y\begin{bmatrix} 11 \\ 7 \\ 3 \end{bmatrix} &= \begin{bmatrix} 26 \\ 12 \\ 2 \end{bmatrix}\end{aligned}$$

The following equations are formed:

$$\begin{aligned}-2x + 11y &= 26 \qquad \ldots (1) \\ x + 7y &= 12 \qquad \ldots (2) \\ 2x + 3y &= 2 \qquad \ldots (3)\end{aligned}$$

Add (1) and (3)

$$14y = 28$$
$$\therefore\ y = 2$$

Substitute in (3)

$$2x + 6 = 2$$
$$\therefore\ x = -2$$

Equation (2) must be checked

$$-2 + 14 = 12$$

Therefore $-2\boldsymbol{a} + 2\boldsymbol{b} = \boldsymbol{d}$.

c $p\boldsymbol{a} + q\boldsymbol{b} - r\boldsymbol{c} = \boldsymbol{0}$

From parts **a** and **b**

$$\boldsymbol{a} + 2\boldsymbol{b} - \boldsymbol{c} = \frac{1}{2}\boldsymbol{d} \qquad \ldots (1)$$
$$-2\boldsymbol{a} + 2\boldsymbol{b} = \boldsymbol{d} \qquad \ldots (2)$$

From (1) $\quad 2\boldsymbol{a} + 4\boldsymbol{b} - 2\boldsymbol{c} = \boldsymbol{d}$

Therefore from (2)

$$-2\boldsymbol{a} + 2\boldsymbol{b} = 2\boldsymbol{a} + 4\boldsymbol{b} - 2\boldsymbol{c}$$
$$\therefore\ 4\boldsymbol{a} + 2\boldsymbol{b} - 2\boldsymbol{c} = \boldsymbol{0}$$

Hence $p = 4, q = 2$ and $r = 2$. (Other answers are possible e.g. $p = 2, q = 1, r = -1$)

4 a $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$$= \begin{bmatrix} 5 \\ 8 \end{bmatrix} + \begin{bmatrix} 20 \\ -15 \end{bmatrix}$$
$$= \begin{bmatrix} 25 \\ -7 \end{bmatrix}$$

The coordinates of Q are $(25, -7)$.

$$\overrightarrow{QR} = \overrightarrow{QO} + \overrightarrow{OR}$$
$$= \begin{bmatrix} -25 \\ 7 \end{bmatrix} + \begin{bmatrix} 32 \\ 17 \end{bmatrix}$$
$$= \begin{bmatrix} 7 \\ 24 \end{bmatrix}$$

b $\overrightarrow{RS} = \overrightarrow{QP}$

$$= \begin{bmatrix} -20 \\ 15 \end{bmatrix}$$

$$\overrightarrow{OS} = \overrightarrow{OR} + \overrightarrow{RS}$$
$$= \begin{bmatrix} 32 \\ 17 \end{bmatrix} + \begin{bmatrix} -20 \\ 15 \end{bmatrix}$$
$$= \begin{bmatrix} 12 \\ 32 \end{bmatrix}$$

Hence the coordinates of S are $(12, 32)$.

5 a $\overrightarrow{OP} = 4\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 12 \\ 4 \end{bmatrix}$$

The coordinates of P are $(12, 4)$.

b $\overrightarrow{PM} = \overrightarrow{PO} + \overrightarrow{OM}$

$$= \begin{bmatrix} -12 \\ -4 \end{bmatrix} + \begin{bmatrix} k \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} k-12 \\ -4 \end{bmatrix}$$

c

$$|\overrightarrow{OP}| = \sqrt{12^2 + 4^2}$$
$$= \sqrt{160}$$
$$= 4\sqrt{10}$$

$$\text{Now } |\overrightarrow{OM}| = k$$

$$\text{and, from part } \mathbf{b}, \overrightarrow{PM} = \begin{bmatrix} k-12 \\ -4 \end{bmatrix}$$

$$\therefore\ |\overrightarrow{PM}| = \sqrt{(k-12)^2 + 16}$$

For triangle OPM to be right-angled at P, Pythagoras' theorem has to be satisfied.

$$\text{i.e. } |\overrightarrow{OP}|^2 + |\overrightarrow{PM}|^2 = |\overrightarrow{OM}|^2$$
$$\therefore\ 160 + (k-12)^2 + 16 = k^2$$
$$\therefore\ 160 + k^2 - 24k + 160 = k^2$$
$$\therefore\ 24k = 320$$
$$\therefore\ 3k = 40$$
$$\therefore\ k = \frac{40}{3}$$

d If M has coordinates $(9, 0)$ then,

if $\angle OPX = \alpha°, \tan\alpha° = 3$

and if $\angle MPX = \beta°, \tan\beta° = \frac{3}{4}$

$$\therefore\ \text{Angle } \theta = \alpha - \beta$$
$$= \tan^{-1}(3) - \tan^{-1}\left(\frac{3}{4}\right)$$
$$= 34.7°,\ \text{correct to one decimal place}$$